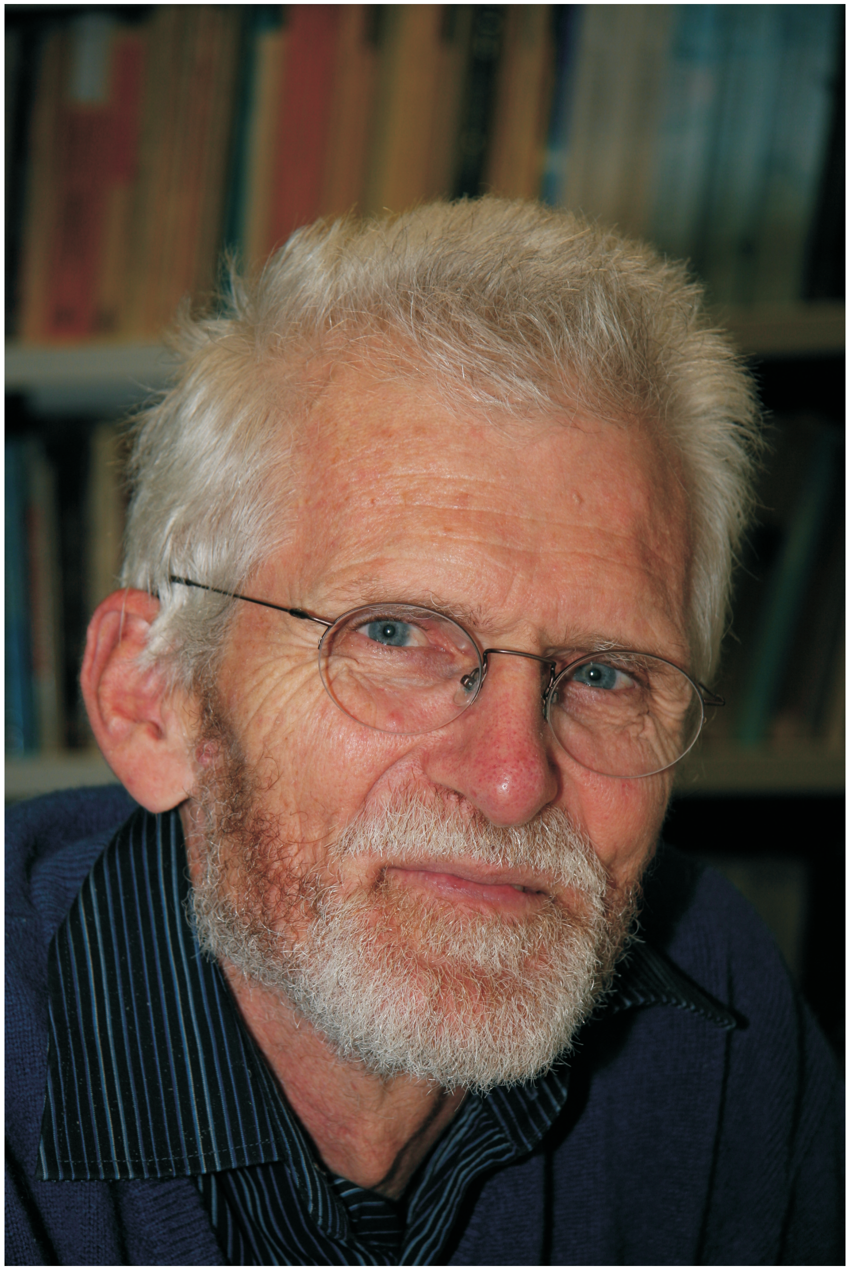

J.J. (Hans) Duistermaat (1942–2010)

Preface

Geometric concepts often play an essential role in obtaining a profound understanding of many areas of analysis and mechanics, for instance, in the theory of Fourier integral operators and in (semi)classical mechanics. This interaction between geometry and analysis or mechanics is a very dominant and also unifying theme in the publications of Hans Duistermaat. At the occasion of his 65th birthday, leading investigators convened at Utrecht University, in August 2007, to discuss recent developments along these lines and in other areas related to the scientific interests of Duistermaat. This volume contains refereed contributions from most of the speakers at this conference and, additionally, two from invited speakers who were unable to attend.

During the preparation of the conference proceedings, Hans Duistermaat passed away unexpectedly, on March 19, 2010. There is no doubt in our minds that Duistermaat would have wished the publication of these proceedings as planned. Accordingly, we decided to leave the format unchanged, but to add an overview of Duistermaat's scientific work as well as some reminiscences by V.W. Guillemin, A. Weinstein, G. Heckman, and R.H. Cushman, as friends and co-authors.

The thirteen research articles published in this volume cover grosso modo three different topics: pseudodifferential operators and (inverse) spectral problems, index theory and localization, and group actions.

Pseudodifferential operators and (inverse) spectral problems. A characterization of the local solvability for square systems of pseudodifferential operators is the topic of the paper of N. Dencker, while J. Sjöstrand describes results on eigenvalue distributions and Weyl laws for non-self-adjoint operators. F. Alberto Grünbaum discusses matrix-valued polynomials satisfying differential equations both with respect to the space and the spectral variables. There are three papers, by S. Vũ Ngọc, Y. Colin de Verdière and V.W. Guillemin, and Y. Colin de Verdière, respectively, on the question to what extent the semiclassical spectrum of an operator determines properties of the operator.

Index theory and localization. In his article, J.-M. Bismut explains the relations between refined versions of index theory on a manifold X and the localization formulas of Duistermaat–Heckman on LX, the associated loop space. P.-E. Paradan studies the local invariants associated to the Hamiltonian action of a compact torus and obtains wall-crossing formulas between invariants attached to adjacent connected components of regular values of the moment map. L. Boutet de Monvel, E. Leichtnam, X. Tang, and A. Weinstein use equivariant Toeplitz operator calculus in order to give a new proof of the Atiyah–Weinstein conjecture on the index of Fourier integral operators and the relative index of CR structures. L.C. Jeffrey and B. McLellan consider the analog of nonabelian localization results of Beasley and Witten when the gauge group G is the abelian group $G = \mathrm{U}(1)$. Finally, E. Meinrenken explains how to define the quantization of q-Hamiltonian SU(2)-spaces as push-forwards in twisted equivariant K-homology, and to prove the "quantization commutes with reduction" theorem for this setting.

Group actions. On a symplectic manifold equipped with a Hamiltonian torus action a real locus is defined to be a set of fixed points for an equivariant smooth anti-symplectic involution. J.-C. Hausmann and T. Holm observe that certain cohomological relations between such a real locus and the ambient manifold can be explained in terms of a purely topological structure, rather than a symplectic one. There is a close relationship between Mumford's geometric invariant theory (GIT) in algebraic geometry and the process of reduction in symplectic geometry. F. Kirwan's paper describes ways in which nonreductive compactified quotients, which cannot be treated by means of classical GIT, can be studied using symplectic techniques.

List of all speakers. Nalini Anantharaman (École Polytechnique, Palaiseau), Nicole Berline (École Polytechnique, Palaiseau), Jean-Michel Bismut (Université Paris-Sud), Yves Colin de Verdière (Université Grenoble), Richard Cushman (Utrecht University), Nils Dencker (Lund University), F. Alberto Grünbaum (University of California at Berkeley), Victor Guillemin (Massachusetts Institute of Technology), Tara Holm (University of Connecticut), Frances Kirwan (University of Oxford), Eugene Lerman (University of Illinois at Urbana-Champaign), Jiang-Hua Lu (University of Hong Kong), Eckhard Meinrenken (University of Toronto), Richard Melrose (Massachusetts Institute of Technology), Paul-Emile Paradan (Université Montpellier 2), Reyer Sjamaar (Cornell University), Gunther Uhlmann (University of Washington at Seattle), San Vũ Ngọc (Université Grenoble), Alan Weinstein (University of California at Berkeley).

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Utrecht
August 2010

Erik P. van den Ban
Johan A.C. Kolk

About J.J. Duistermaat

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16. M.V. Ruzhansky, *Singular Fibrations with Affine Fibers, with Applications to the Regularity Theory of Fourier Integral Operators*, 1998
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Hans Duistermaat (1942–2010)

Erik P. van den Ban* and Johan A.C. Kolk*

On March 19, 2010, mathematics lost one of its leading geometric analysts, Johannes Jisse Duistermaat. At age 67 he passed away, after a short illness following a renewed bout of lymphoma the doctors thought they had controlled. "Hans," as Duistermaat was universally known among friends and colleagues, was not only a brilliant research mathematician and inspiring teacher, but also an accomplished chess player, very fond of several physical sports, and a devoted husband and (grand)father. The remembrances and surveys that follow are from some of his many colleagues, students, and friends. We hope that they adequately convey the impressive breadth of Hans's life and work.

Hans Duistermaat was born December 20, 1942, in The Hague. After the end of World War II his parents moved to the Netherlands East Indies (Indonesia nowadays), where he spent a happy youth. Hans was a student at Utrecht University, where he wrote his Ph.D. thesis on mathematical structures in thermodynamics. The famous geometer Hans Freudenthal is listed as his advisor, but the topic was suggested and the thesis directed by Günther K. Braun, professor in applied mathematics, who tragically died one year before the defense of the thesis, in 1968.

Hans dropped the subject of thermodynamics, because the thesis had led to dissent between mathematicians and physicists at Utrecht University. Nevertheless, this topic exerted a decisive influence on his further development: in its study, Hans had encountered contact transformations. These he studied thoroughly by reading S. Lie, who had initiated their theory. In 1969–1970 he spent one year in Lund, where L. Hörmander was developing the theory of Fourier integral operators (FIOs); these are far-reaching generalizations of partial differential operators. Hans's knowledge of the work of Lie turned out to be an important factor in the formulation of this theory. Hans's mathematical reputation was firmly established by a long joint article with Hörmander concerning applications of the theory to linear partial differential equations. In 1972 Duistermaat was appointed full professor at the

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Catholic University of Nijmegen, and in 1974 at Utrecht University, as the successor to Freudenthal.

In these years, he continued to work on FIOs. At the Courant Institute in New York he wrote a paper on *Oscillatory integrals, Lagrange immersions and unfolding of singularities*, a survey of the subjects in the title that sets the agenda for the study of singularities of smooth functions and their applications to distribution theory. In some sense it is complementary to FIOs and parallel to work of V.I. Arnol'd. Furthermore, together with V.W. Guillemin he composed an article about application of FIOs to the asymptotic behavior of spectra of elliptic operators, and its relation to periodic bicharacteristics; see the article by Guillemin for more details. In these works one clearly discerns the red thread connecting most of Hans's achievements: on the basis of a complete clarification of the underlying geometry deep and powerful results are obtained in the area of geometric analysis.

It is characteristic for Hans's work that after a period of intense concentration on a particular topic, he would move to a different area of mathematics, bringing thereby acquired insights quite often to new fruition. Usually, this change was triggered by a question of a colleague, but more frequently of one of his Ph.D. students. Hans went to great efforts to accommodate the special needs of his students and help them develop in their own way, not in his way. In particular, in several cases Hans was willing and also able to guide students working on topics initiated by themselves. Examples are the theses of P.H.M. van Mouche and M.V. Ruzhansky.

It was by questions of J.A.C. Kolk and G.J. Heckman that Hans became interested in the theory of semisimple Lie groups. With Kolk and V.S. Varadarajan he published fundamental papers on harmonic analysis and the geometry of flag manifolds, with the method of stationary phase as the underlying theme. This work also provided an impetus for the ground-breaking work with Heckman that culminated in the Duistermaat–Heckman formula, which will be discussed separately by Heckman.

In the thesis of E.P. van den Ban one finds the novel idea, suggested by Hans, of taking the integrals representing the spherical eigenfunctions on a semisimple Lie group, which are integrals over a real flag manifold, into integrals on real cycles inside the complex flag manifold. This allowed application of the method of steepest descent in order to study their asymptotics, generalizing the approach known in the theory of hypergeometric functions.

One of Hans's basic mathematical interests, to which he returned throughout his life, was classical mechanics and its relations with differential equations. In this case too, it was often through the work of his students S.J. van Strien, H.E. Nusse, J.C. van der Meer, J. Hermans, B.W. Rink, and A.A.M. Manders that this topic was taken up again. His activities in this area will be further discussed by his colleague and co-author R.H. Cushman.

F.A. Grünbaum posed a problem that led to the joint article *Differential equations in the spectral parameter*. It classifies second-order ordinary differential operators of which the eigenfunctions also satisfy a differential equation in the spectral parameter. The classification is in terms of rational solutions of the Korteweg–de Vries equation.

Writing a review of the book *Lie's Structural Approach to PDE Systems* by O. Stormark led Hans to further study of that circle of ideas. The result was a paper on the contact geometry of minimal surfaces as well as the thesis of P.T. Eendebak.

Together with A. Pelayo he wrote several papers about symplectic differential geometry; furthermore, he directed the thesis of R. Sjamaar. In this part of mathematics Hans was a very influential figure: witness his frequent contacts with other leading investigators, such as Guillemin and A. Weinstein.

In the later part of his life, Hans had an intense interest in applications of mathematics elsewhere in society. For instance, he was a consultant to Royal Dutch Shell, which led to the thesis of C.C. Stolk on the inversion of seismic data. Interaction with mathematical economists during a conference at Erasmus University in Rotterdam, where Hans had been invited to give an introduction to Riemannian geometry, sparked his interest in barrier functions, used in convex programming. He also collaborated with the geophysicist P. Hoyng in modeling the polarity reversals of the earth's magnetic field. The lengths of the time intervals between the subsequent reversals form an irregular sequence with a large variation, which make the reversals look like a (Poisson) stochastic process. Within a short period of time he mastered the nontrivial stochastics needed in this problem.

The bibliography of Hans's work contains eleven books. *Fourier Integral Operators* gives an exposition of seminal results in the area of microlocal analysis. *The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator* is concerned with a direct analytic proof of the index theorem of Atiyah–Singer in a special case of interest for symplectic differential geometry. *Lie Groups*, jointly with Kolk, contains a new proof of Lie's third theorem on the existence of a Lie group associated to any Lie algebra. The construction of the group as the quotient of a path space in the Lie algebra was the model for many important generalizations, including the integration of Lie groupoids by M. Crainic and R.L. Fernandes.

Analysis of Ordinary Differential Equations (in Dutch), jointly with W. Eckhaus, grew out of a set of lecture notes. Similarly, together with Kolk he authored *Multidimensional Real Analysis I: Differentiation* and *II: Integration* (also published in a China edition), and *Distributions: Theory and Applications*. The last book contains a novel proof of the kernel theorem of L. Schwartz, which in turn is used to efficiently derive numerous important results, and a treatment of theories of integration and of distributions from a unified point of view. The last four books together form a veritable "cours d'analyse mathématique."

In the book *Discrete Integrable Systems: QRT Maps and Elliptic Surfaces*, QRT (= Quispel, Roberts, and Thompson) maps are analyzed using the full strength of Kodaira's theory of elliptic surfaces. A complete and self-contained exposition is given of the latter theory, including all the proofs. Many examples of QRT maps from the literature are analyzed in detail, with explicit formulas and computer pictures. The interest in QRT maps was triggered by interaction with J.M. Tuwankotta. Hans had the idea to use the technique of blowing up, which he had previously encountered in the article *Constant terms in powers of a Laurent polynomial* jointly with Wilberd van der Kallen.

While Hans clearly exerted a substantial influence on mathematics through his own research and that of his many Ph.D. students, the books written by him alone or jointly traverse a wide spectrum of mathematical exposition, both in topic or level of sophistication. But in this case again, there is a common characteristic: every result, how hackneyed it may be, had to be fully understood and explained in its proper context. In addition to this, when writing, he insisted that the original works of the masters be studied. Frequently he expressed his admiration for the depth of their treatment, but he could also be quite upset about incomplete proofs that had survived decades of careless inspection. The last project that he was involved in exemplifies this: in joint work with Nalini Joshi reliable proofs are provided of old but also many new results concerning Painlevé functions.

The mode of writing preferred by Hans was top-down exposition: starting from the general, descending to the more concrete. Yet, hidden under the façade of a polished and sometimes quite abstract exposition, there usually was a detailed knowledge of explicit and representative examples. Many of the notebooks he left are filled with intricate calculations, which he performed with great precision and unflagging concentration. Not surprisingly, he greeted the advent of formula manipulation programs like *Mathematica* with great enthusiasm. Furthermore, Hans put a high value on correct illustrations; in private, he could express annoyance about misleading or ugly pictures. In the days of the programming language Pascal and matrix printers, he spent a substantial amount of time in order to put a dot exactly at the position he wanted: one of his favorite techniques for creating complicated illustrations was by printing just a huge number of dots.

In addition to his patience and powers of concentration, he was capable of grasping the essence of a problem and its solution with lightning speed. When this happened during someone's lecture, he usually mentioned this not critically, but kindly and supportively.

As a teacher, Hans was quite aware that not every student was as gifted as he. Despite the fact that he could ignore all restrictions of time and demanded serious work from the students, he was very popular among them. Repeatedly he gave unscheduled courses on their request. He was an honorary member of A-Eskwadraat, the Utrecht Science Students' Society. He shared this honor with Nobel laureate G. 't Hooft and with J.C. Terlouw, a nuclear physicist who pursued a successful career in Dutch politics.

As an administrator, however, he was less successful. Although he served our institute, the mathematical community, and the Royal Netherlands Academy of Arts and Sciences in many different capacities, he was at his best with concrete issues that could be solved rationally, not with situations that required intricate political maneuvering. For instance, he was very actively involved with the Scientific Programme Indonesia–Netherlands, which was an initiative of the academy, aimed at the selection and training of new researchers, the improvement of the supervising infrastructure at Indonesian institutes, and the conduct of joint research activities. In addition, the task of refereeing manuscripts was taken very seriously by Hans: many authors greatly benefited from his long e-mails. He was a member of a substantial number of selection committees, devoting considerable energy to evaluating the candidates' achievements and potential.

In 2004, Hans was honored with a special professorship at Utrecht University endowed by the Royal Netherlands Academy of Arts and Sciences. This position allowed him to focus exclusively on his research, without being distracted by administrative obligations. The five years that followed were a happy period in which his mathematics blossomed. Hans demonstrated by the breadth and depth of his accomplishments that his chair was aptly named "pure and applied mathematics."

His mood was almost invariably one of equanimity; even in difficult situations, he always tended to look for positive aspects. Immense concentration on a topic of momentary interest was natural for him. In fact, on several occasions he confessed that he had a "one-track mind," which made it necessary to mentally exclude disturbances. At times, however, this trait of character could be infuriating for his colleagues.

Very remarkably, Hans had no personal vanity, neither in human nor in professional relations. About his own work he once expressed that he considered himself lucky for having become well known for results he considered to be relatively simple. Most of his more difficult work, which had been far more difficult to achieve, had not received similar recognition. Honors did not mean much to Hans, although he was at first surprised and then gratified by them. He gave himself without any reservation to his friends and colleagues, always illuminating whatever was under discussion with characteristic insights based on his wide knowledge of mathematical and other topics.

In mathematics, Hans's life was a search for exhaustive solutions to important problems. This quest he pursued with impressive single-mindedness, persistence, power, and success. We know that this is a very sketchy attempt to bring him to life. In our minds, however, he is very vivid, one of the most striking among the mathematicians we have met. We deeply mourn his loss; yet we can take comfort in memories of many years of true and inspiring friendship.

Recollections of Hans Duistermaat

Victor W. Guillemin*

The two paragraphs below are a few brief recollections of mine from the period 1973–1974, the two years in which Hans Duistermaat and I worked together on our article "The spectrum of positive elliptic operators and periodic bicharacteristics" (and for me the two most memorable and exciting years of that decade). In the summer of '73, Hans and I met for the first time at an AMS-sponsored conference on differential geometry at Stanford and began to formulate the ideas that became the wave trace part of our paper. Then in the fall of 1974 he made a long visit to MIT, during which we firmed up these ideas and also proved the periodic bicharacteristic results that became the second main part of our paper.

A little prehistory: In the early 1970s, Bob Seeley, David Schaeffer, Shlomo Sternberg, and I ran a seminar at Harvard which was largely devoted to Hörmander's papers [1] and [2] and Hörmander–Duistermaat [3]. In particular, we spent a lot of time going through [3], which was the first systematic application of microlocal techniques to the problem of propagation of singularities. (Like analysts the world over, we were amazed at how simple this subject becomes when viewed from the perspective of the cotangent bundle.) Therefore, when I met Hans that summer I was well primed to discuss with him the contents of these papers. However, what initiated our collaboration was another memorable event from that conference: the announcement by Marcel Berger of Yves Colin de Verdière's result on the spectral determinability of the period spectrum of a Riemannian manifold. I vividly remember sitting next to Hans at Berger's lecture and our exchanging whispered comments as it became more and more evident that what Yves had done was intimately related to the things the two of us were currently thinking about. By the time the conference ended we had formulated a trace theorem for FIOs which asserted that the singularities of the wave trace are supported on the period spectrum of P (and hence that the wave trace gives one a simple means of accessing these data). As I mentioned above, this result became the first part of our paper. The second part was based on

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an observation that Hans and I had made (each independently) that arose apropos of a result of Hörmander's in [1]. One of the most quoted results of Hörmander's paper is a generalization of a theorem of Avakumovic in which he obtains an "optimal" error term in the Weyl law for an elliptic pseudodifferential operator P and shows that this error term is indeed optimal by showing that this is the case if P is the Laplace operator on the standard round sphere. I noticed that this can be related to the fact that for the n-sphere the bicharacteristic flow associated with P is periodic. (More explicitly, I noticed that if the bicharacteristic flow of an elliptic operator P is periodic (i.e., P is Zoll) there has to be a clustering of eigenvalues about a lattice which prevents a sharpening of the Weyl law and vice versa.) In proving this result I made essential use of techniques developed in [3], so it was not surprising that when I described it to Hans at Stanford, I found that he had been thinking along similar lines. Moreover, it slowly began to dawn on us that the Hörmander example was just the tip of the iceberg. Among other things we noticed that his optimal error term could be replaced by a slightly better optimal error term (a big "O" could be converted into a little "o") if P was not Zoll, and also noticed that in this case the Weyl law could be differentiated to give an equidistribution result for eigenvalues. We also obtained a much sharper version of my clustering result: we showed that the clusters are clearly demarcated eigenbands of fixed width. Subsequently Alan Weinstein and Yves Colin de Verdière added a further dimension to this story by discovering that when Zoll phenomena are present, these clusters satisfy their own beautiful distribution law. Furthermore, Bill Helton discovered an extremely clean and economical version of our result: Let A be set of numbers obtained by taking all differences of pairs of eigenvalues, and let B be the cluster set of A. Then if the bicharacteristic flow is periodic, B is an integer lattice and if not, it is the whole real line.

At any rate, to conclude these reminiscences, by the spring of 1974, most of the conjectures we had made at the Stanford meeting had been supplied with rigorous proofs, although Hans continued, as was his wont, to tinker with them for the next several months just to make sure that they were "best possible." (No one was going to be able to achieve instant immortality by slightly improving them.) When Hans visited me in the fall the only unfinished piece of business was the Zoll part of the paper, and that consumed all our energies for four intense weeks. (One typical "Hans" memory from that time: One evening I return home late in the evening exhausted in mind and spirit following a frustrating day in which the two of us struggled without success to settle a delicate point about how large sets of periodic bicharacteristics have to be for clustering to occur. At 2 o'clock in the morning I get jarred awake by a phone call from Hans letting me know that he'd settled it.) I remember the aftermath of Hans's visit as a period of a slow, painful decompression. Never before had I worked so intensely and so single-mindedly on a project (and, for better or for worse, was destined never to do so again).

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Recollections of Hans Duistermaat

Alan Weinstein

I first met Hans in the fall of 1972; my notes from his lecture in Princeton on "Non-involutive operators" continue to make good reading. His work on ordinary differential equations was already well known to me; his study of periodic orbits for the spring pendulum was the inspiration for the thesis of my first PhD student, Jair Koiller.

Hans and his familty then stayed in Berkeley in the summer of 1973 while we were attending the AMS Summer Institute in differential geometry at Stanford. I still have a picture in my mind of our daughters, aged about 2 at the time, playing in the sandbox in our backyard. As for mathematics, that was the time when contact with Hans deepened my interest in Fourier integral operators. Although Hans was not a co-author of Part 1 of the illustrious pair of papers by Hörmander, his influence is clear (and he is the only person thanked by Hörmander in that article).

We met again at a 1974 meeting in Nice, and then spent a lot of time together at the 1975 Nordic Summer School in Grebbestad, Sweden. Here I was totally immersed in the world of microlocal analysis (and Hans was also immersed in the nearby sea, which was too cold for anyone but him and the Finns in our group). A Google search for Duistermaat and Grebbestad turns up exactly two results—links to Hans's famous paper on global action-angle coordinates and my own rather obscure one on the order and symbol of a distribution.

This was just the beginning of Hans's influence on me through his papers and our frequent meetings. Other important influences were the 1972 NYU Lecture Notes on Fourier Integral Operators, which I was very pleased to incorporate later on into the Progress in Mathematics book series, where it remains one of the best places to learn about this subject. Nowhere else is the symplectic geometry of this subject, including the geometry of the Maslov class, so beautifully and concisely explained. The book is also notable for a section at the end linking the homogeneous and "asymptotic" theories of microlocal analysis.

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The wonderful 1975 work with Guillemin on spectrum and periodic bicharacteristics extended the work of Colin de Verdière and Chazarain and set the paradigm for how such matters should be treated. During a visit by Hans to Berkeley, we tried to understand how the Birkhoff normal form of a periodic orbit might be encoded in the spectrum, but that project unfortunately remained unfinished.

A series of papers with Kolk and Varadarajan (1979, 1983) treated harmonic analysis on noncompact semisimple Lie groups. Hans's interest in Lie groups also led to the book with Kolk (2000), based on a course which Hans taught at Berkeley, among other places, and for which a manuscript circulated for many years before publication. Their beautiful proof of Lie's third theorem, constructing a Lie group as a quotient of the paths in the Lie algebra, suggested to me that there should be a similar construction going from Lie algebroids to groupoids. This was carried out in fundamental work of Crainic and Fernandes whose influence continues to this day.

I last saw Hans in the summer of 2009, when I was in Utrecht for a PhD thesis defense. I have a nice photo of the two of us in academic garb in the garden of the cloister at the university. The photo was taken by Marius Crainic, but the setting was carefully managed by Hans. Hans told me that he would be having a surgery later that summer but did not sound particularly concerned; he was obviously hiding the worst from me.

The remarks above cover only a tiny part of Hans's life and work, but they show that both his personal influence and "higher-order effects" have left a lasting mark on mathematicians and mathematics. It was a shock to lose him so suddenly, and his presence in our world will be sorely missed.

Recollections of Hans Duistermaat

Gert Heckman*

I would like to share with you some recollections of Hans Duistermaat from the period 1978–1981, during which he played a crucial role in my mathematical development. In 1976 I had started my dissertation work under the guidance of Gerrit van Dijk. In his thesis of 1962, the Russian mathematician Alexander Kirillov had developed a very elegant geometric method, the so-called orbit method, for understanding the representation theory of connected nilpotent Lie groups. In this method the branching rule for understanding how an irreducible representation decomposes under restriction to a subgroup has a very simple and elegant answer.

Gerrit suggested to me that I try to understand to what extent this orbit method could shed new light on the representation theory of semisimple Lie groups, in particular for the discrete series representations. In my first short paper from the summer of 1978 I worked out a particular example for compact Lie groups. According to the customs of those days I sent it around to several potentially interested people, and in return quickly received a reaction from Hans. My main result turned out to be already in the literature, and in addition Hans sketched an alternative and more elegant geometric proof. Aware of the fact that his letter might be intimidating for me he wrote at the end: “It is maybe superfluous to emphasize that I do not write you this proof out of pedantry, but rather as a sign of interest for your work, and I do hope that it leads to a still better understanding of the whole situation.”

So my first little paper went into the wastebasket, but I was to receive something more valuable in return. I visited Hans regularly in Utrecht, and in June 1980 I defended my dissertation in Leiden with both Gerrit and Hans as thesis advisors. I realize now how lucky I was to have these two complementary teachers: Gerrit with his extensive knowledge of the work of Harish-Chandra, and Hans as the eminent analyst and geometer.

In August 1980 I went to Boston, to spend two years as a postdoc at MIT, and in September I lectured in the Lie groups seminar about my thesis work: how the

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orbit method for compact Lie groups describes the branching rules in an asymptotic way, and how this leads to a convex polytope in which the multiplicities of the branching rule have their support [9]. The talk was received well, most notably by Victor Guillemin. Victor knew Hans well, and had great admiration for him. In 1975 they had written a beautiful article on the spectrum of elliptic operators on compact manifolds [5]. Looking back at my time at MIT, I realize again how lucky I was to be there during that period with Victor around.

That fall a number of new insights were unveiled regarding continuous symmetry reduction in symplectic geometry through the work of Guillemin–Sternberg, Atiyah–Bott, and Mumford. In these at first sight rather different contexts, namely quantum mechanics, quantum field theory, and algebraic geometry, there was a single fundamental underlying concept for the description of symmetry, namely that of the geometry of the moment map (or momentum map as Hans preferred to call it). I quote from a survey article by Bott from 1988 [4]:

> In fact, it is quite depressing to see how long it is taking us collectively to truly sort out symplectic geometry. I became aware of this especially when one fine afternoon in 1980, Michael Atiyah and I were trying to work in my office at Harvard. I say trying, because the noise in the neighboring office made by Sternberg and Guillemin made it difficult. So we went next door to arrange a truce and in the process discovered that we were grosso modo doing the same thing. Later Mumford joined us, and before the afternoon was over we saw how Mumford's stability theory fitted with the Morse theory. The important link here is the concept of a moment map, which in turn is the mathematical expression of the relation between symmetries of Lagrangians and conserved quantities; in short, what the physicists call Noether's theorem and which is one of their great paradigms.

In this quotation Bott refers to the results of fundamental publications by Guillemin–Sternberg [7], [8], Mumford [12], Ness [13], Atiyah–Bott [1], and Kirwan [10]. Since then, symplectic geometry has become a truly independent field in its own right.

In the spring of 1981 Victor gave a course on symplectic geometry, with special emphasis on the geometry of the moment map, and I learned the subject well. During the month of August I went back to the Netherlands to visit family and friends. The day before my return I was doing some last-minute work at MIT, when it occurred to me that the rather complicated locally polynomial formulas for the multiplicities could be explained by a linear variation of the symplectic form in the cohomology of the reduced phase space, at least over the generic fiber. A nice idea, but I had no clue how to prove it. A few days after my return I visited Hans, and we spent a whole afternoon talking about symplectic geometry. I told him about my question, and he listened attentively. That same evening he called me up at my parents' house, and with a piece of scratch paper on my lap I got an exposition of what later would become our joint paper [6].

Our work was well received. Independently of one other, Berline–Vergne [3] and Atiyah–Bott [2] placed it in the more general framework of equivariant cohomology. Our article was used later by Ed Witten in his work on two-dimensional Yang–Mills theory [14]. More recently our theorem was used again by Mariyam Mirzakhani in her computation of the Weil–Petersson volumes of the moduli space of curves [11].

In September 1982 I obtained a permanent position in Leiden as an assistant to Gerrit van Dijk: a solid base from which to pursue mathematical work professionally. I now appreciate very well the important role played by Hans during the early stages of my career. It is not inconceivable that without him I would have become a high-school teacher rather than a university professor of mathematics.

After this period of intensive contact from 1978 to 1981 our mathematical roads diverged. Our personal relationship remained, however, and I cherish the memories of the parties held for his 60th birthday and on the occasion of his royal decoration.

The sudden passing of Hans leaves behind a great emptiness, in the first place for his wife Saskia, his daughters Kim and Maaike and his relatives, but also for the many mathematicians with whom he collaborated. During the cremation ceremony many affecting words were spoken about Hans. His sister Dineke told the story of how, when she asked him as a student why he had chosen mathematics, Hans replied that he had no other option, because his talent for mathematics was such a godsend. I realize how very lucky I am that Hans shared this talent with me so generously.

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Classical mechanics and Hans Duistermaat

Richard H. Cushman*

One of Hans's favorite subjects was classical mechanics. As can be seen from his list of publications, his interest in this area was wide-ranging. In order to describe what he did, I will organize these papers, somewhat arbitrarily, into three classes: (i) periodic solutions near an equilibrium point; (ii) monodromy in integrable systems; (iii) other topics.

In [1] Hans studied the persistence of periodic solutions near an equilibrium point of a two-degrees-of-freedom Hamiltonian system which is in 1 : 2 resonance. This is the simplest situation in which a well-known theorem of Lyapunov on the persistence of periodic solutions fails. Years later Hans returned to this subject in the almost forgotten paper [4]. Here, using the theory of singularities of mappings which are invariant under a circle action that fixes the origin, he proved a stability result for the set of short-period periodic orbits near an equilibrium point of a resonant Hamiltonian system of two degrees of freedom. In particular, he showed that this set of periodic orbits is diffeomorphic to the set of critical points of rank one of the energy-momentum mapping. Here the energy is the Hamiltonian of the Birkhoff normal form of the original resonant Hamiltonian truncated at some finite order. The momentum of the circle action is the quadratic terms of this normal form. As far as I am concerned, this result is the definitive generalization of Lyapunov's theorem.

Hans's most important contribution to the geometric study of Hamiltonian systems is his discovery of the phenomenon of monodromy in [3]. To describe what monodromy is we look at a two-degrees-of-freedom Hamiltonian system on four-dimensional phase space, which we assume is Euclidean space. We suppose that this Hamiltonian system has another function, which is an integral, that is, is constant on the motions of the original Hamiltonian system. Such a Hamiltonian system is said to be completely integrable with integral map given by assigning to each point in phase space the value of the Hamiltonian and the extra integral. If we assume that the integral map is proper and each preimage of a point is connected, then

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the action-angle theorem shows that the preimage of a suitably small open 2-disk in the set of regular values of the integral map is symplectically diffeomorphic to a product of a 2-torus and 2-disk. Hans showed that this local theorem need not hold globally. In particular, if we have a smooth closed, nonintersecting curve in the set of regular values of the integral map, then the preimage of this curve in phase space under the integral map is the total space of a 2-torus bundle, which need not be diffeomorphic to a product of the closed curve and a 2-torus. To understand what this global twisting is, we note that a 2-torus bundle over a circle may be looked at as a product bundle over a closed interval with a typical fiber a 2-torus. Here each of its two end 2-tori, which are Euclidean 2-space modulo the lattice of points with integer coordinates, are glued together by an integer 2×2 matrix with determinant 1. The monodromy of this 2-torus bundle is just this integer matrix. If the monodromy is not the identity matrix, then the 2-torus bundle is not a product bundle. In [3] Hans gave a list of geometric and analytic obstructions for local action-angle coordinates to be global. Monodromy is just the simplest obstruction. Monodromy would not be interesting if there were no two-degrees-of-freedom integrable Hamiltonian systems having it. When Hans was starting to write [3], he asked me to find an example of such a system. The next day I told him that the spherical pendulum, which was studied by Christiaan Huygens in 1612, has monodromy and gave him a proof. When writing up the paper Hans found a much simpler geometric argument to show that the spherical pendulum has monodromy. In [6] Hans and I discovered that monodromy appears in the joint spectrum of the energy and angular momentum operators of the quantized spherical pendulum. This discovery has now been recognized as fundamental by chemists who study the spectra of molecules and has led to a very active area of scientific research. In the early days, showing that a particular integrable system had monodromy was not easy. In [8] Hans did this for the Hamiltonian Hopf bifurcation.

In the middle 1990s Hans became interested in nonholonomically constrained systems such as the disk or a dynamically symmetric sphere with its center of mass not at its geometric center. Both are assumed to be rolling without slipping on a horizontal plane under the influence of a constant vertical gravitational force. This interest gave rise to [9]. In this paper Hans gave a simple geometric criterion for a not necessarily Hamiltonian system to have monodromy. He showed that an oblate ellipsoid of revolution rolling without slipping on a horizontal plane under the influence of a constant vertical gravitational force has a cycle of heteroclinic hyperbolic equilibria whose local monodromies add up to the identity. This shows that it cannot be made into a Hamiltonian system. The book [11] clearly indicates Hans's contributions to the geometric study of nonholonomically constrained systems. Especially, it contains a complete qualitative study of the motion of the rolling disk, some of which was published in [10].

His remaining publications range from removing the incompleteness of the flow of the Kepler problem for all negative energies at the same time, see [7], to showing that the $1:1:2$ resonance is nonintegrable by looking at its fourth-order normal form. In two degrees of freedom, integrability cannot be decided by any finite-order normal form. In the remaining paper on periodic linear Hamiltonian systems [2]

Hans answered an old question of Bott's about the Morse index of iterates of a periodic geodesic. Bott showed that this index is the sum of the index of the periodic geodesic and invariants of the real symplectic conjugacy class of the linear Poincaré map. Hans gave an explicit formula for the Morse index.

Working with Hans and collaborating on our joint publications is at the core of my mathematical career. It is hard for me to realize that he cannot answer my questions anymore.

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